

The chemical evolution of galaxies with a variable IGIMF

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ABSTRACT

Standard analytical chemical evolution modelling of galaxies has been assuming the stellar initial mass function (IMF) to be invariant and fully sampled allowing fractions of massive stars to contribute even in dwarf galaxies with very low star formation rates (SFRs). Recent observations show the integrated galactic initial mass function (IGIMF) of stars, i.e. the galaxy-wide IMF, to become systematically top-heavy with increasing SFR. This has been predicted by the IGIMF theory, which is here used to develop the analytical theory of the chemical evolution of galaxies. This theory is non-linear and requires the iterative solution of implicit integral equations due to the dependence of the IGIMF on the metallicity and on the SFR. It is shown that the mass–metallicity relation of galaxies emerges naturally, although at low masses the theoretical predictions overestimate the observations by 0.3–0.4 dex. A good agreement with the observation can be obtained only if gas flows are taken into account. In particular, we are able to reproduce the mass–metallicity relation observed by Lee et al. (2006) with modest amounts of infall and with an outflow rate which decreases as a function of the galactic mass. The outflow rates required to fit the data are considerably smaller than required in models with invariant IMFs.

Key words: Stars: abundances – stars: luminosity function, mass function – supernovae: general – Galaxies: evolution – Galaxies: dwarf – Galaxies: star clusters: general

1 INTRODUCTION

The stellar initial mass function (IMF) has been traditionally interpreted to be an invariant probability density distribution function (e.g. Kroupa 2001, 2002; Bastian et al. 2010). Recent observational evidence has however ruled out this simple interpretation in that low-density regions have been found to be lacking the massive stars they ought to have produced given the large number of stars formed (Kirk & Myers 2011; Hsu et al. 2012). The mass function of young star clusters has also been shown to not be a probability density distribution function (Pflamm-Altenburg et al. 2013; Kroupa 2014). Furthermore, several independent observational evidences have been put forward in the last few years, showing that the galaxy-wide IMF varies with star formation rate (SFR) (e.g. Hoversten & Glazebrook 2008; Gunawardhana et al. 2011; Cappellari et al. 2012; Conroy & van Dokkum 2012) and probably varies also within a single galaxy (Dutton et al. 2013).

A comprehensive theory, attempting to explain the evo-

lution of the IMF within galaxies is the so-called integrated galactic IMF (IGIMF, i.e. the galaxy-wide IMF) theory (Kroupa & Weidner 2003; Weidner & Kroupa 2005; see the recent review by Kroupa et al. 2013). In two previous papers (Recchi et al. 2009; Recchi et al. 2014) we have investigated the effect of the IGIMF on the abundance ratios (global and of individual stars) in galaxies. One paper (Calura et al. 2010) has been devoted to the study of the chemical evolution of the Solar Neighborhood by adopting the (SFR-dependent) IGIMF formulation. Eventually, numerical hydrodynamical simulations of dwarf galaxies with a variable IGIMF have been pioneered, too (Ploechinger et al. 2014). It remains to be shown that the IGIMF theory is consistent with the observed mass-metallicity (MZ) relation in galaxies (see e.g. Lee et al. 2006; Kirby et al. 2013) and with the more general fundamental metallicity relation (FMR, see Mannucci et al. 2010; Lara-López et al. 2010).

We build on a previous attempt (Köppen et al. 2007) and solve the equations of the so-called “Simple Model” but within the framework of the IGIMF theory, i.e. we assume that the galaxy-wide IMF depends on the SFR and on the metallicity of the parent galaxy. Analytical and semi-analytical calculations of the chemical evolution of galaxies

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based on simple models are nowadays quite popular, as they enlighten in a simple way complex correlations among galaxies (Spitoni et al. 2010; Dayal et al. 2013; Lilly et al. 2013; Birrer et al. 2014; Pipino et al. 2014). It is worth remarking that all these authors consider invariant IMFs whereas, as explained above, there is significant observational evidence for IMF variations among different galaxies. Therefore, it is important and timely to understand how to modify the formalism of the simple model to take into account IMF variations. This is indeed the main aim of the present paper. It is also worth stressing that, although this paper is based on the IGIMF theory, other theories of IMF variation have been put forward in the literature (e.g. Larson 1998; Padoan & Nordlund 2002; Hopkins 2013. See also Martinelli & Matteucci 2000). The formalism developed in this paper can be applied equally well to these IMF theories.

The paper is organized as follows: in Sect. 2 we recall the main assumptions of the IGIMF theory and describe the way in which this theory has been implemented in the present work. In Sect. 3 we describe the simple models of chemical evolution – a simplified tool to describe the evolution of the metallicity of a galaxy. In Sect. 4 we re-interpret the so-called closed-box model within the framework of the IGIMF theory, i.e. we relax the hypothesis that the IMF is universal. Similarly, in Sect. 5 we re-interpret simple models of chemical evolution with outflows and infall and consider again variations of the IMF between galaxies and within galaxies. Finally, in Sec. 6 the results of this work are discussed and some conclusions are drawn. In a companion paper (Recchi et al. 2014; hereafter Paper II), we will apply this methodology to the study of the mass-metallicity relation in tidal dwarf galaxies (TDGs).

2 THE IGIMF THEORY

The IGIMF theory is based on the assumption that stars in a galaxy are formed in space-time correlated entities called embedded star clusters (Lada & Lada 2003), thereby accounting naturally for the above mentioned results by Kirk & Myers (2011), Hsu et al. (2012) and Pflamm-Altenburg et al. (2013). The notion underlying the IGIMF theory lies in the conjecture that the freshly born galaxy-wide stellar population is the sum of all local star-formation activity and that this activity occurs in the density maxima of molecular cloud cores by being made up of embedded groups or clusters of stars on a spacial scale of less than a pc (Marks & Kroupa 2012). The mass distribution of these is equivalent to the mass function of embedded clusters which do not need to be gravitationally bound upon expulsion of their residual gas (Lada et al. 1984; Kroupa et al. 2001; Banerjee & Kroupa 2013). An “isolated” or low-density mode of star formation is naturally accounted for in this picture by low-mass and/or the radially extended part of the embedded clusters.

In defining the IGIMF theory one posits the following six axioms derived from observations (see Kroupa et al. 2013; Weidner et al. 2013)¹

- (i) The stellar IMF, $\xi(m)$, in an embedded star cluster is

canonical (a two-part power law $\xi(m) \propto m^{-\alpha_i}$ with $\alpha_1 = 1.3$ for $0.08 M_\odot < m < 0.5 M_\odot$, and $\alpha_2 = 2.3$ for stellar masses larger than $0.5 M_\odot$) for cloud core densities $\rho_{\text{ecl}} \leq 9.5 \times 10^4 M_\odot \text{ pc}^{-3}$. A star formation efficiency $\epsilon = M_{\text{ecl}}/(M_{\text{ecl}} + M_{\text{gas}})$ in molecular cloud cores of 0.33 has been assumed (Alves et al. 2007). Here M_{ecl} is the stellar mass in the embedded cluster and M_{gas} is the residual gas mass within it.

- (ii) The embedded clusters populate an embedded cluster mass function (ECMF), which is assumed to be a power law of the form $M_{\text{ecl}} \propto M_{\text{ecl}}^{-\beta}$ ($\beta = 2.35$ would be the Salpeter index).

- (iii) The half-mass radii of embedded clusters follow $r_h (\text{pc}) = 0.1 M_{\text{ecl}}^{0.13}$ (Marks & Kroupa 2012) yielding for the total density (gas plus stars) $\log_{10}(\rho_{\text{cl}}) = 0.61 \log_{10}(M_{\text{ecl}}) + 2.85$, in units of $M_\odot \text{ pc}^{-3}$.

- (iv) The most massive star in a cluster, m_{max} , is a function of the stellar mass of the embedded cluster, M_{ecl} (Weidner & Kroupa 2004, 2006; Weidner et al. 2010), $m_{\text{max}} = m_{\text{max}}(M_{\text{ecl}}) \leq m_{\text{max}*} \approx 150 M_\odot$ (e.g. eq. 10 in Pflamm-Altenburg et al. 2007).

- (v) There exists a relation between the SFR of a galaxy and the most massive young (age < 10 Myr) star cluster, $\log_{10}(M_{\text{ecl}}) = 0.764 \log_{10}(\text{SFR}) + 4.93$ (Weidner et al. 2004), where the SFR is in units of $M_\odot \text{ yr}^{-1}$. The mass of the least-massive embedded “cluster” is assumed to be $5 M_\odot$.

- (vi) The dependence of the IMF slope, α_3 , of stars above $1 M_\odot$ on the initial density and metallicity of the embedded cluster is given by (Marks et al. 2012)

$$\alpha_3 = \begin{cases} 2.3 & \text{if } x < -0.87 \\ -0.41 \times x + 1.94 & \text{if } x \geq -0.87 \end{cases}, \quad (1)$$

where

$$x = -0.14[\text{Fe}/\text{H}] + 0.99 \log_{10} \left(\frac{\rho_{\text{cl}}}{10^6 M_\odot \text{ pc}^{-3}} \right). \quad (2)$$

This dependency of the stellar IMF on density and metallicity of a massive star-burst cluster has been derived by combining independent evidence from globular clusters (Marks et al. 2012) and ultra-compact dwarf galaxies (Dabringhausen et al. 2009, 2012).

For a value of $\beta = 2$ the IGIMF built in this way nicely reproduces many observed properties of galaxies (see Kroupa et al. 2013). It also qualitatively reproduces the trend of decreasing α_3 with increasing SFR observed by Gunawardhana et al. (2011). In order to improve this fit, a seventh axiom has been introduced (Weidner et al. 2013): the exponent β of the ECMF is not a constant but it varies with the SFR according to:

$$\beta = \begin{cases} 2.00 & \text{if } \text{SFR} < 1 M_\odot \text{ yr}^{-1}, \\ -0.106 \log_{10}(\text{SFR}) + 2.00 & \text{if } \text{SFR} \geq 1 M_\odot \text{ yr}^{-1}. \end{cases} \quad (3)$$

Axiom 7 implies the mass function of embedded clusters to be top-heavy in star bursts. Weidner et al. (2013) explored also the possibility that the mass of the smallest embedded cluster depends on the SFR, thus partially relaxing the fifth axiom above. They showed however that this assumption produces results almost indistinguishable from the results obtained with axiom 7, thus we will not further consider this hypothesis.

¹ Throughout this text m and M_{ecl} are in units of M_\odot .

3 SIMPLE MODELS OF CHEMICAL EVOLUTION: THE INVARIANT IMF CASE

A class of galactic chemical evolution models, commonly dubbed as Simple Models, admit analytical solutions. They are thus quite useful and easy-to-apply diagnostics for the evolution of galaxies (see Spitoni et al. 2010; Lilly et al. 2013; Pipino et al. 2014; see also the Introduction), thus it is useful to understand advantages and limitations of this formalism.

The basic assumptions behind the simple models are (see Tinsley 1980; Maeder 1992):

- (i) Stars more massive than a fixed mass (usually $1 M_{\odot}$) die instantaneously. Stars less massive than this threshold live forever. This assumption (instantaneous recycling approximation) allows us to neglect the lifetime of single stars in what follows.
- (ii) The gas is well mixed at any time.
- (iii) The flow rates (infall and outflow) are proportional to the SFR ψ .
- (iv) The IMF is universal: it is independent of time and it is the same in all galaxies.
- (v) The yields and the remnant masses are functions of the stellar mass alone. This assumption (together with the universality of the IMF) implies that the quantities:

$$R = \int_1^{\infty} (m - m_R) \xi(m) dm, \quad (4)$$

$$yz = \frac{1}{1 - R} \int_1^{\infty} m p_{Zm} \xi(m) dm,$$

are constant. Here, ξ is the adopted (fixed, for the moment) IMF, m_R is the remnant mass of the star with initial mass m , and p_{Zm} is the mass-fraction of newly synthesised metals by the star of initial mass m . According to this terminology, the total amount of heavy elements released by a star of initial mass m , initial metallicity Z and remnant mass m_R is $m_{ej,Z} = m p_{Zm} + (m - m_R)Z$, i.e. it includes newly synthesised heavy elements but also metals present in the stars at birth and not processed. The quantity R in Eq. 4 represents thus the fraction of a stellar population not locked into long-living (dark) remnants and yz represents the ratio between the mass of heavy elements ejected by a stellar generation and the mass locked up in remnants.

It is well known that these assumptions lead to a set of differential equations for the time evolution of the total baryonic mass M_t , total gas mass M_g and total mass in metals M_Z within a galaxy (see Tinsley 1980; Maeder 1992; Pagel 1997; Matteucci 2001):

$$\begin{cases} \frac{dM_t}{dt} = (\Lambda - \lambda)(1 - R)\psi(t) \\ \frac{dM_g}{dt} = (\Lambda - \lambda - 1)(1 - R)\psi(t) \\ \frac{dM_Z}{dt} = (1 - R)\psi(t)[\Lambda Z_A + yz - (\lambda + 1)Z] \end{cases} \quad (5)$$

Here, Z_A is the metallicity of the infalling material, whereas Λ and λ are proportionality constants relating the SFR to the infall and outflow rate, respectively. In particular, it is assumed that:

$$A(t) = \Lambda(1 - R)\psi(t), \quad W(t) = \lambda(1 - R)\psi(t), \quad (6)$$

(see Matteucci & Chiosi 1983) where $A(t)$ and $W(t)$ are the infall or accretion and the outflow rate, respectively. Given the initial condition $M_t(0) = M_g(0) = M_0$, $Z(0) = Z_0$, a

solution as a function of the mass ratio $\mu = M_g/M_t$ can be found (see also Recchi et al. 2008):

$$Z = Z_0 + \frac{\Lambda Z_A + yz}{\Lambda} \left\{ 1 - \left[(\Lambda - \lambda) - \frac{\Lambda - \lambda - 1}{\mu} \right]^{\frac{\Lambda}{\Lambda - \lambda - 1}} \right\}. \quad (7)$$

This solution expresses the gas-phase metallicity of a galaxy, as a function of the mass fraction μ . The time dependence of this relation is hidden in the time dependence of μ , which in turn depends on how quickly the gas is consumed to form stars (i.e. it depends on the SFR) and on the flows in and out of the galaxy. Hence, to have a complete picture of the evolution of the galaxy, we need also some assumption concerning the SFR. In what follows we will assume that the SFR depends on the available gas reservoir through the simple law

$$\psi = s \cdot M_g, \quad s = 0.3 \text{ Gyr}^{-1}, \quad (8)$$

i.e. we assume here that the SFR is linearly proportional to the gas mass and that this proportionality constant is an invariant quantity. This equation also implies that the gas consumption timescale of a galaxy is always about 3 Gyr. Notice that many studies of the chemical evolution of galaxies suggest that the star formation efficiency increases with the mass of the galaxy (e.g. Matteucci 1994; Pipino & Matteucci 2004). These studies are however based on invariant IMFs and indeed Matteucci (1994) showed that a variable IMF or a variable star formation efficiency might produce similar results. In the framework of the IGIMF, Pflamm-Altenburg & Kroupa (2009) showed that the gas consumption time-scale is similar in all galaxies, irrespective of their masses, and it is indeed of the order of 3 Gyr. We consider also the average metallicity Z_* of the stars in a galaxy. A simple way of calculating this quantity is:

$$Z_* = \frac{\int_0^t Z(t)\psi(t)dt}{\int_0^t \psi(t)dt}. \quad (9)$$

This expression represents the mass-weighted average of the metallicities of all the stellar populations ever born in the galaxy (see Pagel 1997). A more accurate definition of Z_* would involve a luminosity-weighted average, but it has been shown that these two averages do not differ much (e.g. Recchi et al. 2009).

A solution can be found also in the presence of metal-enhanced galactic outflows (see Sect. 5). A solution can also be obtained assuming generic infall and outflow laws (see again Recchi et al. 2008). From the given solution Eq. 7, other particular cases can be found. For instance, setting $\Lambda = \lambda = \alpha = Z_A = 0$ one obtains the well-known solution of the closed-box model $Z = -yz \ln(\mu)^2$

² In Eq. 7 a Λ is in the denominator, hence $\frac{\Lambda Z_A + yz}{\Lambda}$ diverges as $\Lambda \rightarrow 0$. The previous sentence might thus look suspicious. The fact is that also the expression within the curly brackets tends to zero for $\Lambda \rightarrow 0$. Take $\lambda = Z_A = 0$; the solution reduces to

$$Z = \frac{yz}{\Lambda} \left[1 - \left(\Lambda - \frac{\Lambda - 1}{\mu} \right)^{\frac{\Lambda}{\Lambda - 1}} \right] \simeq \frac{yz}{\Lambda} \left[1 - \left(\frac{1}{\mu} \right)^{-\Lambda} \right], \quad (10)$$

where we have taken the limit for small Λ in the second equality. Write $\left(\frac{1}{\mu} \right)^{-\Lambda}$ as $e^{-\Lambda \ln(1/\mu)}$ and expand it in a Taylor series, e.g. $e^{-\Lambda \ln(1/\mu)} \simeq 1 - \Lambda \ln(1/\mu)$. By substituting this expansion in the

It is generally accepted that this solution gives a first-order accurate description of the overall evolution of chemical elements (such as the α -elements) mainly released by massive stars on short timescales.

4 THE CLOSED-BOX MODEL REVISITED: THE VARIABLE IGIMF

The relevant equations for the closed-box model with a linear SFR such as in Eq. 8 are:

$$\begin{cases} \frac{dM_*}{dt} = 0 \\ \frac{dM_g}{dt} = -(1-R)\psi(t) \\ \frac{dM_Z}{dt} = (1-R)\psi(t)[y_Z - Z] \\ \psi(t) = sM_g(t) \end{cases} \quad (11)$$

It is trivial to solve for $Z(t)$, $M_g(t)$. Assuming that there is only gas at the beginning and that $Z(0) = 0$, one obtains:

$$Z(t) = s(1-R)y_Z t, \quad M_g(t) = M_g(0) \exp[-s(1-R)t]. \quad (12)$$

It is important to remark on this equation that, according to the assumptions of the simple model, all constants that enter in the calculation of $Z(t)$ are indeed constants, i.e. they do not depend on the mass of the model galaxy taken into consideration. The consequence is that the time evolution of the metallicity is always the same, independent of the mass of the galaxy. Consequently, these models can not reproduce the observed MZ relation. A viable solution is to assume that small galaxies can experience galactic winds, due to their shallow potential wells. Observations show indeed that mass-loading factors in galactic winds are larger in small galaxies (Martin 2005) and theoretical investigations confirm this finding (Hopkins et al. 2012; Puchwein & Springel 2013). If the parameter λ introduced in Sect. 3 is large in small galaxies and small or zero in large ones, the MZ relation can be nicely reproduced (Tremonti et al. 2004; Spitoni et al. 2010).

Although we are convinced that galactic winds can play an important role in the chemical evolution of dwarf galaxies (Recchi & Hensler 2013; see also Sect. 5), we want to explore here the possibility that the IMF is not constant but that it depends on time though its dependence on the SFR and on Z , as per the IGIMF theory described in Sect. 2. As explained in the Introduction, this exploration is purely didactic and serves to illustrate in the simplest possible setting how a variable IMF affects the formalism of the simple model described in Sect. 3.

Clearly, the assumption of a variable IMF introduces a differentiation in the metallicity evolution of model galaxies with different masses, as the SFR of different model galaxies are different and that implies variations in the IGIMF, which in turn imply variations in the chemical evolution.

To be more quantitative, the quantities R and y_Z introduced in Eq. 4 become:

$$\begin{aligned} R(t) &= \int_1^\infty (m - m_R) \xi_{\text{IGIMF}}[m, \psi(t), Z(t)] dm, \\ y_Z(t) &= \frac{1}{1-R} \int_1^\infty m p_{Zm} \xi_{\text{IGIMF}}[m, \psi(t), Z(t)] dm, \end{aligned} \quad (13)$$

i.e. they are not constant any more, but are functions of time, through the time dependence of ψ and Z . The IMF ξ_{IGIMF} entering here is the IGIMF, i.e. the IMF obtained adopting the axioms described in Sect. 2. The formal solution of the closed-box equations 11 is now:

$$\begin{aligned} Z(t) &= Z_0 + s \int_0^t [1 - R(\tau)] y_Z(\tau) d\tau, \\ M_g(t) &= M_g(0) \exp \left[-s \int_0^t [1 - R(\tau)] d\tau \right], \\ \psi(t) &= sM_g(t) = sM_g(0) \exp \left[-s \int_0^t [1 - R(\tau)] d\tau \right]. \end{aligned} \quad (14)$$

We have included also the solution for $\psi(t)$ to show that this system of equations is indeed implicit (the integrals in the right hand sides depend on the quantities on the left hand sides). To solve these equations thus, an implicit procedure must be employed.

Examples of the calculation of the yields y_Z using the IGIMF theory and the axioms described in Sect. 2 are shown in Fig. 1. In particular, the left panel refers to a model (Model 1) in which the axioms 1–6 of Sect. 2 are adopted. The right panel instead refers to Model 2, i.e. a model where also the last axiom of Sect. 2 is implemented. As mentioned in the Introduction, the methodology described in this paper is not limited to the IGIMF theory. Other theories of variable IMF could be used to calculate y_Z and R as a function of galactic properties. Once y_Z and R have been calculated, the rest of the theory is general and not related to a specific IMF formulation.

To construct these functions $y_Z(t)$, we have adopted the yields of Woosley & Weaver (1995) for massive stars and the ones of van den Hoek & Groenewegen (1997) for intermediate-mass stars. Both sets of yields are metallicity-dependent. Since we are interested in the early phases of the evolution of galaxies, we have thus taken the set of yields with $Z=0.001$ from van den Hoek & Groenewegen (1997) and the one with $Z=10^{-4} Z_\odot$ from Woosley & Weaver (1995). We have checked that our results depend little on the chosen metallicity, although larger initial metallicities tend to produce slightly higher yields. The increase of y_Z with SFR is due to the fact that the IGIMF is flatter for high SFRs, thus more metals are produced. Because of Eq. 1 (and because of the dependence of x on $[\text{Fe}/\text{H}]$), the yield increases also with decreasing metallicity. The yields of Model 2 are slightly larger than the yields of Model 1, because of the axiom 7 of Sect. 2. The differences between Model 1 and Model 2 however disappear for SFRs smaller than $1 M_\odot \text{ yr}^{-1}$ because of Eq. 3. Given the small differences between Model 1 and Model 2, we will focus from now on on results pertaining Model 1. Notice also that the yields tend to flatten out at very low SFRs. This is due to the fact that low- and intermediate-mass AGB stars still produce some metals according to the yields of van den Hoek & Groenewegen and the fraction of AGB stars (in particular the fraction of low-mass AGB stars) changes very little with the SFR. The

expression for Z in Eq. 10, the closed box solution $Z = y_Z \ln(1/\mu)$ is recovered.

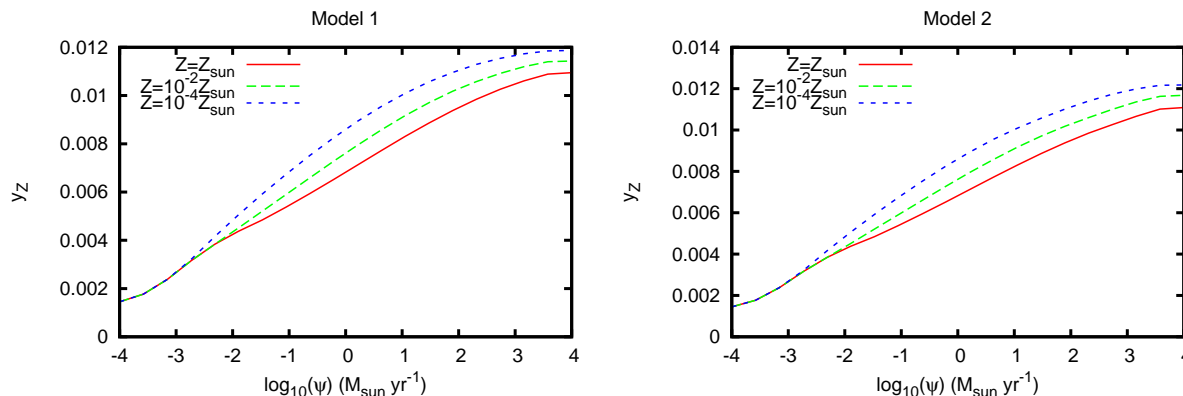


Figure 1. The yield y_Z , as defined in Eq. 13, as a function of the SFR ψ and of the metallicity. The left panel refers to the Model 1 (axioms 1–6 of Sect. 2) and the right panel refers to Model 2 (where also the last axiom of Sect. 2 is implemented). In comparison, an invariant IMF has $y_Z = \text{constant}$ (Sec. 3).

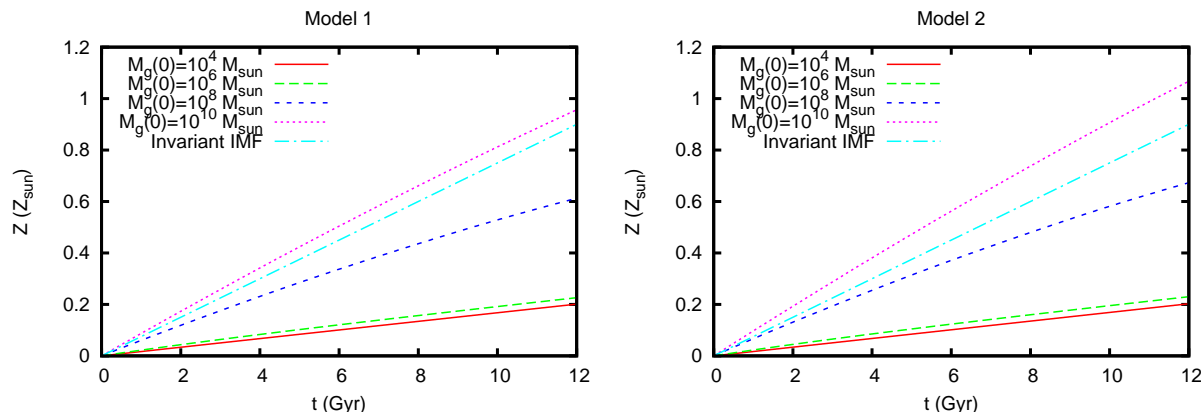


Figure 2. The gas-phase metallicity Z as a function of time for four representative model galaxies with different initial gas masses, from $10^4 M_\odot$ (lower curves) to $10^{10} M_\odot$ (upper curves). The left panel refers to Model 1 (axioms 1–6 of Sect. 2) and the right panel refers to Model 2 (where also the last axiom of Sect. 2 is implemented). In each panel a model with an invariant IMF (thus with no dependence on the initial gas mass of the model) is shown for comparison.

function $R(\psi)$ is qualitatively similar to the shown function $y_Z(\psi)$, although the magnitude of the variations is smaller.

Fig. 2 depicts the time evolution of four representative model galaxies with different initial gaseous masses, ranging from $10^4 M_\odot$ (lower curves) to $10^{10} M_\odot$ (upper curves). The left panel refers to the Model 1 (axioms 1–6 of Sect. 2) and the right panel refers to Model 2 (where also the last axiom of Sect. 2 is implemented). In these models it is assumed that all the mass initially is in the form of gas. Clearly, the metallicity evolution depends now, at variance with the invariant IMF case, on the model galaxy, as larger galaxies have on average higher SFR, hence flatter IMFs, with a higher production rate of heavy elements. As a comparison, a model with an invariant IMF is shown in Fig. 2, too. Typical parameters ($y_Z = 0.01$ and $R = 0.5$) have been used to plot these curves. It is true that low-metallicity models tend to have flatter IMFs, too (see Eq. 1 and the definition of the parameter x), but since the proportionality constant in front of $[\text{Fe}/\text{H}]$ in the definition of x is small, the metallicity effect on the IMF is outweighed by the SFR effect. Due to the larger yields, the final metallicities for Model 2 are (slightly) larger than the ones for Model 1.

For all these model galaxies, we have also calculated the average stellar metallicities, according to Eq. 9. In principle, these stellar metallicities could be compared with the best observational data available to us, i.e. the stellar metallicities of dwarf satellites of the Milky Way and in general of dwarf galaxies of the Local Group. The stellar metallicities of these galaxies are well determined and they build a clear and tight MZ relation (see e.g. Kirby et al. 2013). Unfortunately, the $[\text{Fe}/\text{H}]$ abundance is usually measured in stars (see again Kirby et al. 2013). This makes the comparison with simple model predictions much less straightforward. In fact, as it is well known, the majority of iron is produced by Type Ia Supernovae, over long timescales. As recalled in Sect. 3, the simple models give acceptable results only for elements (such as the α -elements) produced on short timescales by massive stars. Hence, simple models adopting the instantaneous recycling approximation cannot predict the right Fe abundance since they cannot take into account properly the contribution from Type Ia SNe. We are working on an extension of the simple model formalism that takes into account also iron produced by SNeIa. At the moment, results of simple models can be safely compared to

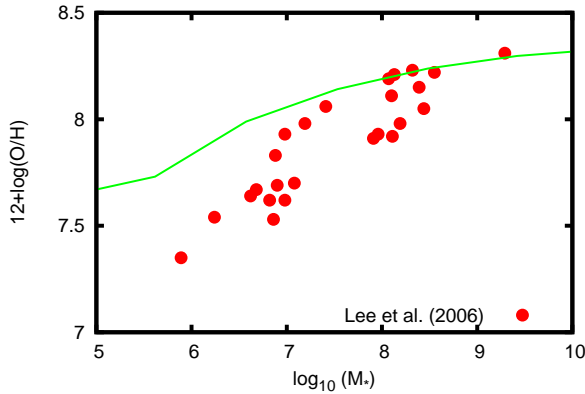


Figure 3. The MZ relation obtained at an age of 12 Gyr by means of the simple closed box model within the IGIMF theory adopting Model 1 (solid line; Model 2 is barely distinguishable from Model 1 as is evident in Fig. 2.). The Lee et al. (2006) observations are shown as red circles.

gas-phase abundances of α -elements only, and this is what we do in Fig. 3. We use the data of Lee et al. (2006) and we compare them with the results of model galaxies shown in Fig. 2. The initial metallicity considered here (the Z_0 in Eq. 14) is zero. Initial metallicities larger than zero are important when studying TDGs, therefore they will be considered extensively in Paper II. We anticipate however here that, if TDGs form at very large redshifts out of galaxies whose chemical enrichment is still limited, their initial metallicity is not large and $Z_0 = 0$ is a reasonable assumption.

Clearly, we can see a MZ in Fig. 3, in the sense that low-mass models attain smaller stellar metallicities than high-mass ones. However, the comparison with observations is not particularly good. In particular at low masses the theoretical predictions overestimate the observations by 0.3–0.4 dex. This is due in part to the inherent uncertainties in the Lee et al. (2006) MZ relation, in part to the fact that galactic flows in low-mass systems can not be neglected. We analyze in detail the effect of galactic flows in the following section.

5 THE LEAKY BOX MODEL REVISITED

We revise now the system of equations 11 to take into account possible gas inflows and outflows in galaxies. As already mentioned, this will be done by introducing parameters Λ and λ that relate the flow rates to the SFR (see Eq. 6). Additionally, we consider the possibility that the outflows are metal-enriched (*differential winds*), i.e. that the metallicity of the outflowing gas is larger than the average gas-phase metallicity in the galaxy. Both observations (Martin et al. 2002; Ott et al. 2005) and numerical models (Mac-Low & Ferrara 1999; Recchi & Hensler 2013) support this assumption. Notice that probably also the efficiency with which the various chemical species are ejected vary from element to element (*selective winds*; see Recchi et al. 2001; 2008) but for simplicity we will not consider further this hypothesis.

Assuming that the metallicity of the outflowing gas is $Z_o = \gamma Z$, with $\gamma \geq 1$, we can write:

$$\begin{cases} \frac{dM_t}{dt} = (\Lambda - \lambda)(1 - R)\psi(t), \\ \frac{dM_g}{dt} = (\Lambda - \lambda - 1)(1 - R)\psi(t), \\ \frac{dM_Z}{dt} = (1 - R)\psi(t)[\Lambda Z_A + y_Z - (\gamma\lambda + 1)Z], \\ \psi(t) = sM_g(t). \end{cases} \quad (15)$$

The solution of this system of equations for $M_g(t)$ and $Z(t)$ is given by:

$$\begin{aligned} M_g(t) &= M_g(0) \exp \left[(\Lambda - \lambda - 1)s \int_0^t [1 - R(\tau)] d\tau \right], \\ Z(t) &= Z_0 + \frac{\int_0^t [1 - R(\tau)] s [\Lambda Z_A + y_Z(\tau)] I(\tau) d\tau}{I(t)}, \\ I(t) &= \exp \left[- \int_0^t [1 - R(\tau)] s [\lambda(\gamma - 1) + \Lambda] d\tau \right] \end{aligned} \quad (16)$$

Here, $I(t)$ is an appropriate integration factor. Notice that the expression for $Z(t)$ reduces to the analogous closed-box expression shown in Eq. 14 in the case $\gamma = 1$, $\Lambda = 0$ (i.e. no differential winds, no infall).

As already mentioned, it is to be expected that low-mass galaxies are much more affected by galactic winds than are more-massive ones, i.e. we expect λ to decrease as the galactic mass increases. A simple functional form satisfying this condition is³:

$$\lambda = 2 \cdot 10^{1-0.2 \log_{10} M_g(0)}. \quad (17)$$

This formula is normalized in order to reproduce the outflow rate to SFR ratio in TDG simulations in which the IGIMF prescriptions are adopted (Ploekinger et al. 2014). According to this formula, λ tends to be quite large for very low mass systems (it is $\lambda \simeq 3.17$ for $M_g(0) = 10^4 M_\odot$) but it tends to zero for very high mass ones (it is $\lambda \simeq 0.13$ for $M_g(0) = 10^{11} M_\odot$). On the other hand, we do not expect the infall to significantly depend on the mass of the parent galaxy, therefore we take, in compliance with Spitoni et al. (2010), $\Lambda = 0.5$ for all model galaxies. Notice that the values of λ are considerably smaller than the ones needed in order to reproduce the MZ relation based on an invariant IMF. The comparison between our adopted values of λ and the ones of Spitoni et al. (2010) for $\Lambda = 0.5$ are given in Fig. 4. Notice that we are interested in this study in smaller systems compared to the ones considered by Spitoni et al. (2010), therefore the two curves in Fig. 4 do not fully overlap. It is nevertheless clear that the outflow rates in Spitoni et al. (2010) are much larger than the ones adopted in this study. This is due to the fact that the reduction of the metallicity at low masses is, according to the IGIMF theory, largely due to the steepening of the IMF slope at large stellar masses in galaxies with small SFRs. Moreover, we assume $\gamma = 2.5$, based on the results of the chemo-dynamical simulations of Recchi & Hensler (2013).

The MZ relation obtained with leaky box models with this dependence of λ with $M_g(0)$ is shown in Fig. 5. The fit with observations is very good. It could be better if we better fine-tune the relation $\lambda = \lambda(M_g(0))$ but this is not the main aim of the paper.

³ Actually, one expects λ to decrease with increasing total (baryons plus dark matter) galactic mass. If we assume for simplicity a constant ratio between dark matter and baryonic mass, a correlation between λ and $M_g(0)$ as in Eq. 17 is recovered.

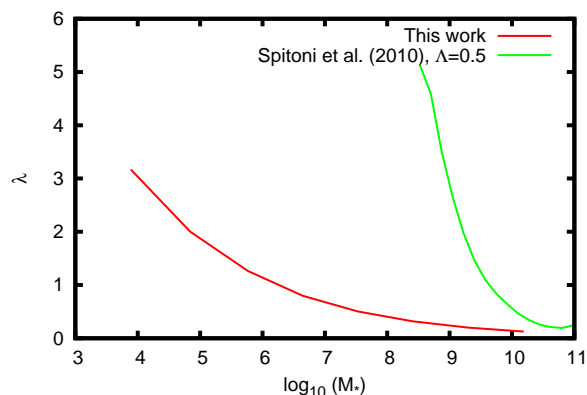


Figure 4. The comparison between the outflow rates λ adopted in this study (see Eq. 17) and the ones adopted for $\Lambda = 0.5$ in Spitoni et al. (2010).

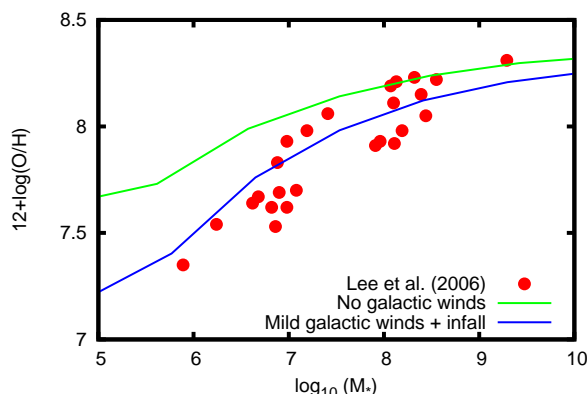


Figure 5. The MZ relation obtained by means of the leaky box model within the IGIMF theory. Λ is assumed to be equal to 0.5 and the correlation between λ and $M_g(0)$ is as in Eq. 17.

6 DISCUSSION AND CONCLUSIONS

The IGIMF theory (Kroupa et al. 2013) predicts a coupling between some properties of a galaxy (the SFR and the metallicity) and the IMF. Since the IMF in turn strongly affects the dynamical evolution of the galaxy, the feedback between galaxy evolution and IMF is difficult and the fully complexity of a variable IMF has not been yet included in hydrodynamical simulations (but see Bekki 2013; Ploekinger et al. 2014; Recchi 2014). Even the here treated approach based on the so-called “simple model of chemical evolution” leads to implicit integral equations that must be solved iteratively. We note in passing that the dependence of the IMF on the metallicity is well established theoretically, as low-metallicity gas cools less efficiently and self-gravitating clumps are more resilient to fragmentation. This leads to the formation of dense cores of higher mass. For this reason, the IMF is supposed to be extremely biased towards massive stars if the metallicity is smaller than $\sim 10^{-4} Z_{\odot}$ (Schneider et al. 2002). Also the dependence of the IMF on the SFR is nowadays observationally well established (Hoversten & Glazebrook 2008; Meurer et al. 2009; Gunawardhana et al. 2011; Kroupa 2014), and the IGIMF theory is the only existing computable access accounting for these observations

(Weidner et al. 2013). Thus, in spite of the inherent complexity, detailed simulations of galaxy evolution based on variable IMFs need to be performed. With this paper, we aimed at showing in a simple setting how to take into account IMF variations in models of the chemical evolution of galaxies (see also Martinelli & Matteucci 2000; Calura et al. 2010). The method outlined here can be used in order to extend analytical studies of the evolution of galaxies based on simple models of chemical evolution (e.g. Spitoni et al. 2010; Lilly et al. 2013; Pipino et al. 2014). We described how the solution of the simple model differs from the standard, textbook analytical solutions of the simple model. We also showed (building on previous results of Köppen et al. 2007) that the IGIMF theory naturally leads to a mass-metallicity relation. In fact, low-mass galaxies are characterized on average by smaller SFRs. According to the IGIMF theory, a low SFR leads to a steep, top-light IMF, in which the production of heavy elements by massive stars is extremely limited. More massive galaxies instead produce many more massive stars because of the higher level of SFR, hence the attained present-day metallicity is larger.

We outline here once again that the main aim of the paper was not to provide the ultimate explanation for the mass-metallicity relation. Many detailed theoretical studies have been performed on this subject (see e.g. Matteucci & Chiosi 1983; Tremonti et al. 2004; Finlator & Davé 2008; Calura et al. 2009; Peeples & Shankar 2011; Yin et al. 2011 among many others). As shown by Fig. 3, the MZ relation obtained with the IGIMF theory does not fit well the data and assumptions about flow rates are required in order to obtain a good fit. Nevertheless it is important to show that (i) The IGIMF theory (as well as any other theory according to which the IMF is top-heavier in more massive systems) naturally accounts for the MZ relation; (ii) Even within the IGIMF formalism, flow rates (infall and outflows) remain crucial ingredients for the evolution of galaxies; (iii) The intensity of the galactic wind must be assumed to be inversely proportional to the initial gaseous mass of the model galaxy, in accordance with observations and theoretical studies. The outflow rates are however considerably smaller than the ones needed to reproduce the MZ relation in models with invariant IMFs such as the one of Spitoni et al. (2010).

In a companion paper this methodology will be applied to the study of the mass-metallicity relation in dark-matter-free tidal dwarf galaxies. At the same time, efforts are ongoing to model by means of hydrodynamical codes the full chemo-dynamical evolution of galaxies within the IGIMF theory, in order to assess the reliability of the results obtained in this paper.

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